

nag_1d_spline_evaluate (e02bbc)

1. Purpose

nag_1d_spline_evaluate (e02bbc) evaluates a cubic spline from its B-spline representation.

2. Specification

```
#include <nag.h>
#include <nage02.h>

void nag_1d_spline_evaluate(double x, double *s, Nag_Spline *spline,
                          NagError *fail)
```

3. Description

This function evaluates the cubic spline $s(x)$ at a prescribed argument x from its augmented knot set λ_i , for $i = 1, 2, \dots, \bar{n} + 7$, (see **nag_1d_spline_fit_knots** (e02bac)) and from the coefficients c_i , for $i = 1, 2, \dots, q$ in its B-spline representation

$$s(x) = \sum_{i=1}^q c_i N_i(x)$$

Here $q = \bar{n} + 3$, where \bar{n} is the number of intervals of the spline, and $N_i(x)$ denotes the normalised B-spline of degree 3 defined upon the knots $\lambda_i, \lambda_{i+1}, \dots, \lambda_{i+4}$. The prescribed argument x must satisfy $\lambda_4 \leq x \leq \lambda_{\bar{n}+4}$.

It is assumed that $\lambda_j \geq \lambda_{j-1}$, for $j = 2, 3, \dots, \bar{n} + 7$, and $\lambda_{\bar{n}+4} > \lambda_4$.

The method employed is that of evaluation by taking convex combinations due to de Boor (1972). For further details of the algorithm and its use see Cox (1972) and Cox (1978).

It is expected that a common use of **nag_1d_spline_evaluate** (e02bbc) will be the evaluation of the cubic spline approximations produced by **nag_1d_spline_fit_knots** (e02bac). A generalization of **nag_1d_spline_evaluate** which also forms the derivative of $s(x)$ is **nag_1d_spline_deriv** (e02bcc). **nag_1d_spline_deriv** (e02bcc) takes about 50% longer than **nag_1d_spline_evaluate**.

4. Parameters

x

Input: the argument x at which the cubic spline is to be evaluated.
Constraint: **spline.lamda**[3] \leq **x** \leq **spline.lamda**[**spline.n**–4].

s

Output: the value of the spline, $s(x)$.

spline

Input: Pointer to structure of type **Nag_Spline** with the following members:

n - Integer

Input: $\bar{n} + 7$, where \bar{n} is the number of intervals (one greater than the number of interior knots, i.e., the knots strictly within the range λ_4 to $\lambda_{\bar{n}+4}$) over which the spline is defined.

Constraint: **spline.n** \geq 8.

lamda - double *

Input: a pointer to which memory of size **spline.n** must be allocated. **spline.lamda**[$j - 1$] must be set to the value of the j th member of the complete set of knots, λ_j for $j = 1, 2, \dots, \bar{n} + 7$.

Constraint: the λ_j must be in non-decreasing order with **spline.lamda**[**spline.n**–4] $>$ **spline.lamda**[3].

c - double *

Input: a pointer to which memory of size **spline.n**–4 must be allocated. **spline.c** holds the coefficient c_i of the B-spline $N_i(x)$, for $i = 1, 2, \dots, \bar{n} + 3$.

Under normal usage, the call to nag_1d_spline_evaluate will follow a call to nag_1d_spline_fit_knots (e02bac), nag_1d_spline_fit_knots (e02bac) or nag_1d_spline_fit (e02bec). In that case, the structure **spline** will have been set up correctly for input to nag_1d_spline_evaluate.

fail

The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_INT_ARG_LT

On entry, **spline.n** must not be less than 8: **spline.n** = $\langle value \rangle$.

NE_ABSCL_OUTSIDE_KNOT_INTVL

On entry, **x** must satisfy $\mathbf{spline.lamda}[3] \leq \mathbf{x} \leq \mathbf{spline.lamda}[\mathbf{spline.n}-4]$:
spline.lamda[3] = $\langle value \rangle$, **x** = $\langle value \rangle$, **spline.lamda**[$\langle value \rangle$] = $\langle value \rangle$.

In this case **s** is set arbitrarily to zero.

6. Further Comments

The time taken by the function is approximately $C \times (1 + 0.1 \times \log(\bar{n} + 7))$ seconds, where C is a machine-dependent constant.

Note: the function does not test all the conditions on the knots given in the description of **spline.lamda** in Section 4, since to do this would result in a computation time approximately linear in $\bar{n} + 7$ instead of $\log(\bar{n} + 7)$. All the conditions are tested in nag_1d_spline_fit_knots (e02bac), however, and the knots returned by nag_1d_spline_interpolant (e01bac) or nag_1d_spline_fit (e02bec) will satisfy the conditions.

6.1. Accuracy

The computed value of $s(x)$ has negligible error in most practical situations. Specifically, this value has an absolute error bounded in modulus by $18 \times c_{max} \times \mathbf{machine\ precision}$, where c_{max} is the largest in modulus of c_j, c_{j+1}, c_{j+2} and c_{j+3} , and j is an integer such that $\lambda_{j+3} \leq x \leq \lambda_{j+4}$. If c_j, c_{j+1}, c_{j+2} and c_{j+3} are all of the same sign, then the computed value of $s(x)$ has a relative error not exceeding $20 \times \mathbf{machine\ precision}$ in modulus. For further details see Cox (1978).

6.2. References

- Cox M G (1972) The Numerical Evaluation of B-splines *J. Inst. Math. Appl.* **10** 134-149.
 Cox M G (1978) The Numerical Evaluation of a Spline from its B-spline Representation *J. Inst. Math. Appl.* **21** 135-143.
 Cox M G and Hayes J G (1973) *Curve Fitting: A Guide and Suite of Algorithms for the Non-specialist User* Report NAC26, National Physical Laboratory, Teddington, Middlesex.
 De Boor C (1972) On Calculating with B-splines *J. Approx. Theory* **6** 50-62.

7. See Also

nag_1d_spline_interpolant (e01bac)
 nag_1d_spline_fit_knots (e02bac)
 nag_1d_spline_deriv (e02bcc)
 nag_1d_spline_fit (e02bec)

8. Example

Evaluate at 9 equally-spaced points in the interval $1.0 \leq x \leq 9.0$ the cubic spline with (augmented) knots 1.0, 1.0, 1.0, 1.0, 3.0, 6.0, 8.0, 9.0, 9.0, 9.0 and normalised cubic B-spline coefficients 1.0, 2.0, 4.0, 7.0, 6.0, 4.0, 3.0.

The example program is written in a general form that will enable a cubic spline with \bar{n} intervals, in its normalised cubic B-spline form, to be evaluated at m equally-spaced points in the interval $\mathbf{spline.lamda}[3] \leq x \leq \mathbf{spline.lamda}[\bar{n}+3]$. The program is self-starting in that any number of data sets may be supplied.

8.1. Program Text

```

Nag_Spline spline;

Vprintf("e02bbc Example Program Results\n");
Vscanf("%*[^\\n]"); /* Skip heading in data file */
while(scanf("%ld",&m) !=EOF)
{
    if (m>0)
    {
        Vscanf("%ld",&ncap);
        ncap7 = ncap+7;
        if (ncap>0)
        {
            spline.n = ncap7;
            spline.c = NAG_ALLOC(ncap7, double);
            spline.lamda = NAG_ALLOC(ncap7, double);
            if (spline.c != (double *)0 && spline.lamda != (double *)0)
            {
                for (j=0; j<ncap7; j++)
                    Vscanf("%lf",&(spline.lamda[j]));
                for (j=0; j<ncap+3; j++)
                    Vscanf("%lf",&(spline.c[j]));
                a = spline.lamda[3];
                b = spline.lamda[ncap+3];
                Vprintf("Augmented set of knots stored in spline.lamda:\n");
                for (j=0; j<ncap7; j++)
                    Vprintf("%10.4f%s",spline.lamda[j],
                        (j%6==5 || j==ncap7-1) ? "\\n" : " ");
                Vprintf("\\nB-spline coefficients stored in spline.c\\n\\n");
                for (j=0; j<ncap+3; j++)
                    Vprintf("%10.4f%s",spline.c[j],
                        (j%6==5 || j==ncap+2) ? "\\n" : " ");
                Vprintf("\\n          x          Value of cubic spline\\n\\n");
                for (r=1; r<=m; ++r)
                {
                    x = ((double)(m-r) * a + (double)(r-1) * b) / (double)(m-1);
                    e02bbc(x, &s, &spline, NAGERR_DEFAULT);
                    Vprintf("%10.4f%15.4f\\n",x,s);
                }
                NAG_FREE(spline.c);
                NAG_FREE(spline.lamda);
            }
            else
            {
                Vfprintf(stderr,"Storage allocation failed. Reduce the \\
size of spline.n\\n");
                exit(EXIT_FAILURE);
            }
        }
        else
        {
            Vfprintf(stderr,"ncap is negative or zero : ncap = %ld\\n",ncap);
            exit(EXIT_FAILURE);
        }
    }
    else
    {
        Vfprintf(stderr,"m is negative or zero : m = %ld\\n",m);
        exit(EXIT_FAILURE);
    }
}
exit(EXIT_SUCCESS);
}

```

8.2. Program Data

e02bbc Example Program Data

9

4

1.00
 1.00
 1.00
 1.00
 3.00
 6.00
 8.00
 9.00
 9.00
 9.00
 9.00
 1.00
 2.00
 4.00
 7.00
 6.00
 4.00
 3.00

8.3. Program Results

e02bbc Example Program Results

Augmented set of knots stored in spline.lamda:

1.0000	1.0000	1.0000	1.0000	3.0000	6.0000
8.0000	9.0000	9.0000	9.0000	9.0000	

B-spline coefficients stored in spline.c

1.0000	2.0000	4.0000	7.0000	6.0000	4.0000
3.0000					

x	Value of cubic spline
1.0000	1.0000
2.0000	2.3779
3.0000	3.6229
4.0000	4.8327
5.0000	5.8273
6.0000	6.3571
7.0000	6.1905
8.0000	5.1667
9.0000	3.0000
